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Wavelet Characterization of Ozone Layer Depletion for Atmospheric Region of Pakistan

M. Ayub Khan Yousuf Zai*, M. Rashid Kamal Ansari, Jawaid Iqbal, Arif Hussain, and Ahmed Hasan

Institute of Space and Planetary Astrophysics and Department of Applied Physics, University of Karachi, Karachi, 75270, Pakistan Institute of Mathematical Sciences, Federal University of Arts, Science and Technology, Karachi, Pakistan

Abstract

Ozone is of considerable importance to mankind because of its role as a shield for the biosphere against the Ultraviolet (UV) radiation from the outer space, in particular from the sun. In the recent years the advancement in the science and technology has increased the understanding of various phenomena of the environment and consequently the predictability of many naturally occurring events. It has been Due to a serious reduction of O3 content in the atmosphere life-forms on the earth are exposed to a new hazard in the form of an increase of harmful solar UV radiation intensity at the sea level. Obviously, this situation calls for an assessment, monitoring and prevention of the incidence of decrease in the O3 concentration. The fluctuating dynamics of stratospheric ozone layer depletion (OLD) for Pakistan atmospheric region has been introduced by mentioning the production and annihilation of ozone. The recent development in the theory and application of wavelets are yielding powerful multi resolution techniques for the analysis of time series and image data. Wavelets are the central idea of a broad framework for displaying and analyzing data. Wavelets possess a simultaneous time–frequency localization that makes them useful in presenting complicated signals. In this communication behaviour of ozone is explained on the basis of ozone profile, effects of periodic, a-periodic change along with effects of monthly and seasonal variations using wavelet analysis.

1. Introduction

Ozone in the stratosphere acts to protect us from harmful ultraviolet radiation from the sun that it absorbs. It has been inspected that a small decrease in the ozone concentration can lead to a large increase in the amount of ultraviolet (UV) reaching Arabian sea specially around wavelength range of 290-320 nm. Excessive UV exposure can cause skin cancers and in large amounts can also harmful to plant growth. The highest risks are in spring at extreme southerly latitudes or to a lesser extent at extreme northerly latitudes, corresponding to the formation of the ozone holes over Antarctic and Arctic polar regions.The ozone layer is also a separate issue from climate change and greenhouse warming. The CFC that cause ozone layer depletion are greenhouse gases, but destruction of the ozone layer reduces the greenhouse effect slightly as the ozone also acts as greenhouse gas.

Conversely, if the ozone destruction were to become serious enough to reduce primary productivity of phytoplankton and forest ecosystems, as a result of UV damage, this would affect climate change. As with many environmental systems, all are interconnected. The ozone (O3) forms from oxygen by reaction under the influence of UV radiation. Shorter wavelength radiation, of wavelength 175 nm or less, has sufficient energy to dissociate oxygen into two separate free oxygen atoms. This occurs at heights of about 50-60- km and results in very little radiation this wavelength penetrating further. The free O atom can then combine with O2 molecule to form Ozone. The ozone forms in this above tropical and equatorial regions where solar radiation is highest, and spreads around the globe to form this layer at between 20 and 26 km in height.

* Corresponding author
E-mail: ayubzai@yahoo.com

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The ozone layer if expressed at standard temperature and pressure would be 3 mm thick, yet it absorbs strongly enough in the Uv region a to act as an almost complete shield against radiation of wavelength less than around 295 nm.

$$O2 \rightarrow 2O$$

Oxygen dissociates in presence of UV radiation

$$O2 + O \rightarrow O3$$

Ozone formation

However, in the presence of certain pollutants, chiefly chlorofluorocarbons (CFC)s, ozone can be destroyed. Free chlorine (Cl) radicals acts to strip O3 of an oxygen atom, forming O2 and ClO that then loses the O to a free oxygen atom to form O2 and Cl again [3]

$O3 + Cl \rightarrow O2 + ClO$ Ozone destruction by Cl radical

$ClO + O \rightarrow Cl + O2$ Cl radical free to act again

The net effect is to transform ozone into O2 while leaving the Cl radical free to promote another reaction thus one molecule of CFC can destroy a large number of ozone molecules.

Nitric Oxide and hydroxyl (OH) radicals can act the same way. The resulting reduction in the ozone concentration depends upon a balance between the processes of formation and annihilation. The series of reactions as shown above that destroy ozone take place mostly readily at low temperatures and so occur mainly over the poles during the spring time. It has been investigated that in the region of stratosphere there is little water vapors and so no cloud. During the Antarctic winter temperatures plummet and the temperature of stratosphere drops below around − 800 C, polar stratospheric clouds can form. These clouds allow the formation of highly reactive free Cl radicals. In the early spring as the first rays of sun strike the clouds the reactions that destroy ozone commence. This results in the so - called ozone hole over the Antarctic that can be measured by means of remote sounding equipment where the atmospheric absorption are observed. The Antarctic ozone hole has occurred every winter for the past two decades with increasing severity. During the summer the ozone is replenished from the tropics. It has been observed that ozone loss is less severe at Arctic because stratospheric temperatures are not usually low enough to allow clouds to form, albeit in recent years an Arctic ozone hole has also formed. Reaching a record low in the year 2000 [4].

While production of the most harmful CFCs is not subject to strict international controls stemming from the Montreal Protocol, they are very long-lived once they reach the stratosphere and still being emitted from many sources. The ozone hole appeared to have deteriorated more rapidly than hoped or expected. It is thought this may be due to a feedback with climate change. Increased CO2 levels in the troposphere hold the Earth's heat at the surface, resulting in lowering the temperatures in the stratosphere. These lowering temperatures promote PSCs that allow OLD more rapidly. Hence the ozone holes have increased more rapidly in both size and level of depletion. Obviously, this situation calls for an assessment, monitoring and prevention of the incidence of decrease in the O3 concentration. In this communication we have studied to quantify the wavelet characteristic for ozone layer depletion. In order to strengthen the concepts of wavelet analysis we have introduced some ideas of Fourier Analysis, Fourier Series, Fourier Transform and fast Fourier transforms (fft) Using the Fourier analysis, seasonal trends in atmospheric, meteorological and environmental data sets can be removed.

2. Fourier Analysis, Fourier series and Fourier Transform

We have introduced the basic idea of the signals then sampling the characteristics for OLD. For this purpose we have used Fourier analysis (also known as spectral analysis, frequency analysis, or harmonic analysis) that uses stationary or detrended data sets with inherent periodicities. The time series can be represented by a superposition of sines and cosines of various amplitudes and frequencies as shown in the table 1. With this finite data set of ozone depths (1960-2002) the range of frequencies that can be discerned is limited on the low end to $f_0 = 1 / N$ (the fundamental frequency) by finite length of the record and on the high end to $f_c = I / 2$ (the *Nyquist frequency*) by the sample rate The f_0 comes from at least one full oscillation. The Nyquist frequency from at least two data points per period to define oscillation. The reciprocal of these frequencies are the periods of the corresponding oscillations. Therefore, it can be seen from the figure.1.It depicts enhancement and depletion in the ozone layer at height ranging from 25-55 km. Above sea level with time The data has been supplied from the year 1960 to 2002.

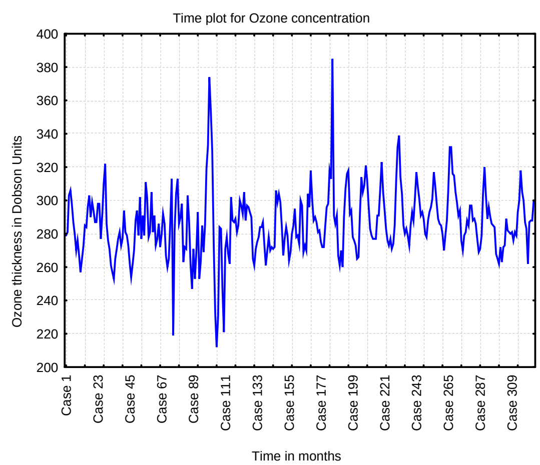

Figure 1. Temporal Fluctuation of Ozone Concentration at Pakistan's Stratosphere

A temporal variation for ozone depths is depicted in fig.1. Thus with a finite data set as mentioned in this paper we can do the best is approximate Xn by

$$Xn = \frac{a_0}{2} + \sum_{m=1}^{N/2} a_m Cos \frac{2\text{®}mn}{N} + b_m \, Sin \frac{2\text{®}mn}{N}$$

Where N /2 different frequencies or harmonics (f = mf0) have amplitudes

$$a_m = \frac{2}{N} \sum_{n=1}^{N} X_n Cos \frac{2\text{®}mn}{N}$$

$$b_m = \frac{2}{N} \sum_{n=1}^{N} X_n Sin \frac{2®mn}{N}$$

These equations constitute the discrete Fourier transform (DFT). The coefficients values as given in the table. with periodogram and density as well as the Hamming values they contain the data points but are in frequency domain rather than the time domain.

Table.1

	Period	Cosine	Sine	Periodog	Density	Hamming
0		-0.00000	0.00000	0.00	946.24	0.035714
1	324.0000	-0.78174	-3.13506	1691.23	1268.12	0.241071
2	162.0000	-1.40758	3.05367	1831.60	1355.94	0.446429
3	108.0000	0.02650	1.38857	312.47	1043.71	0.241071
4	81.0000	1.14279	2.87039	1546.31	1060.83	0.035714
5	64.8000	2.13755	0.72683	825.78	962.25	
6	54.0000	1.14936	1.99612	859.49	671.31	
7	46.2857	0.65010	-0.05822	69.02	417.10	
8	40.5000	-1.41631	0.93554	466.75	562.04	
9	36.0000	-2.13464	-1.36061	1038.09	987.81	
10	32.4000	0.85847	-2.99259	1570.20	1311.15	
11	29.4545	2.30947	-0.01927	864.11	1793.67	
12	27.0000	-1.54234	-4.57637	3778.16	2574.50	
13	24.9231	2.89265	-2.39115	2281.77	2651.09	
14	23.1429	-0.10720	3.55931	2054.18	3008.22	
15	21.6000	-5.51246	-1.84816	5476.08	3753.03	
16	20.2500	3.75735	0.86863	2409.29	3544.89	
17	19.0588	3.60675	3.61439	4223.74	3094.94	
18	18.0000	-0.95564	3.00691	1612.67	2232.03	
19	17.0526	-0.52641	-2.71292	1237.20	1857.39	
20	16.2000	1.91299	3.90875	3067.93	1917.68	
21	15.4286	-1.32188	1.65017	724.21	1312.44	
22	14.7273	-1.40065	-1.04158	493.57	1095.43	
23	14.0870	3.81408	-0.61773	2418.46	1287.70	
24	13.5000	0.42324	1.08949	221.31	824.98	
25	12.9600	0.18765	-1.28943	275.05	1715.93	
26	12.4615	-3.16214	0.48659	1658.22	8016.24	
27	12.0000	1.66582	13.39178	29502.58	14200.78	
28	11.5714	1.82871	-3.48270	2506.69	8445.76	
29	11.1724	-1.48800	0.75601	451.28	2208.53	
30	10.8000	1.13240	2.59775	1300.96	1028.75	
31	10.4516	-2.27032	-0.98937	993.58	885.95	
32	10.1250	-0.86955	-0.99721	283.59	740.41	
33	9.8182	2.64709	-0.80382	1239.82	876.88	
34	9.5294	-0.19506	2.22770	810.11	840.40	
35	9.2571	0.27790	-2.02816	678.88	602.86	
36	9.0000	1.02068	0.08746	170.01	458.22	
37	8.7568	0.42376	1.77300	538.34	740.64	
38	8.5263	0.12191	-3.21683	1678.79	1096.21	
39	8.3077	2.14112	-0.81596	850.53	899.97	
40	8.1000	-0.35863	0.94045	164.12	748.25	
41	7.9024	-3.07491	-0.59824	1589.70	1000.56	
42	7.7143	0.85920	-1.97165	749.35	1009.41	
43	7.5349	-1.08524	2.40543	1128.14	900.59	
44	7.3636	-1.54108	0.11851	387.01	971.90	
45	7.2000	0.29265	-3.37076	1854.52	1357.85	
46	7.0435	3.03355	-0.12055	1493.15	1387.96	
47	6.8936	-2.11664	1.34556	1019.09	986.18	
48	6.7500	0.44153	-1.53776	414.67	543.56	
49	6.6122	0.89054	0.29160	142.25	377.70	
50	6.4800	-1.79396	1.06231	704.18	422.46	
51	6.3529	-1.14986	-0.27166	226.15	325.34	
52	6.2308	-0.87909	0.01307	125.22	283.13	
53	6.1132	-1.60409	-0.87897	542.00	414.36	
54	6.0000	0.20788	1.69652	473.26	493.06	
55	5.8909	1.77681	0.55324	561.03	537.48	
56	5.7857	-1.36426	0.33134	319.30	834.52	
57	5.6842	-2.39626	-2.73815	2144.80	1218.52	
58	5.5862	1.92971	-0.46082	637.65	889.23	

59	5.4915	-1.09701	0.75792	288.02	411.40	
60	5.4000	-0.83639	0.67381	186.88	253.93	
61	5.3115	-0.41686	-1.05551	208.64	370.39	
62	5.2258	1.50926	1.60055	784.02	652.81	
63	5.1429	-2.09387	-1.14202	921.54	768.66	
64	5.0625	1.32889	1.52498	662.83	570.32	
65	4.9846	0.15776	0.38736	28.34	330.60	
66	4.9091	-1.67990	-0.43680	488.09	303.21	
67	4.8358	0.68540	-0.91743	212.45	251.26	
68	4.7647	-0.67047	-0.41454	100.66	212.42	
69	4.6957	-1.50430	-0.25677	377.27	254.86	
70	4.6286	0.11216	-1.16185	220.72	217.30	
71	4.5634	-0.23544	-0.42597	38.37	185.62	
72	4.5000	1.42926	-0.73723	418.98	219.79	
73	4.4384	0.33959	0.15153	22.40	202.70	
74	4.3784	-0.37703	1.27372	285.85	335.17	
75	4.3200	-1.27718	-1.43890	599.66	651.04	
76	4.2632	2.49094	-1.07842	1193.58	895.52	
77	4.2078	-0.75470	1.97602	724.82	859.87	
78	4.1538	-1.99525	1.32324	928.58	656.43	
79	4.1013	-0.44342	-0.60864	91.86	331.28	
80	4.0500	-0.52381	-0.37004	66.63	255.52	
81	4.0000	1.49183	-1.41558	685.17	366.09	
82	3.9512	0.46028	0.83785	148.04	298.64	
83	3.9036	0.09294	0.94133	144.95	335.43	
84	3.8571	-0.62168	-2.19273	841.52	476.10	
85	3.8118	1.00001	-0.57414	215.40	369.50	
86	3.7674	0.39501	1.12786	231.35	260.48	
87	3.7241	-1.26653	0.20460	266.65	259.32	
88	3.6818	1.31426	-0.40274	306.09	241.01	
89	3.6404	0.53077	0.49102	84.70	210.97	
90	3.6000	-	1.35912	319.30	261.79	
		0.35182				
91	3.5604	-1.31670	0.70017	360.28	266.26	
92	3.5217	-0.05268	0.42130	29.20	255.97	
93	3.4839	-0.17642	-1.77390	514.81	393.71	
94	3.4468	1.34941	1.31724	576.08	431.22	
95	3.4105	-0.36982	-0.86255	142.68	322.62	
96	3.3750	-1.05619	-1.17893	405.88	262.39	
97	3.3402	0.66547	-0.46546	106.84	154.64	
98	3.3061	-0.27686	-0.06442	13.09	59.68	
99	3.2727	0.37854	-0.05211	23.65	77.52	
100	3.2400	1.15179	-0.19148	220.85	163.47	
101	3.2079	-0.90088	0.59511	188.85	228.49	
102	3.1765	0.66940	-1.35282	369.07	225.90	
103	3.1456	0.33068	-0.29126	31.46	119.46	
104	3.1154	-0.01686	0.15560	3.97	84.77	
105	3.0857	-0.60566	-1.06968	244.79	142.21	
106	3.0566	0.59595	-0.44862	90.14	176.60	
107	3.0280	1.06362	-0.66758	255.46	253.46	
108	3.0000	-1.01434	1.29254	437.33	289.17	
109	2.9725	-0.76991	-0.11743	98.26	224.26	
110	2.9455	-0.20735	-0.94785	152.51	313.74	
111	2.9189	2.14389	-0.65477	814.05	472.81	
112	2.8929	-1.10705	0.72151	282.87	345.46	
113	2.8673	-0.26322	-0.30180	25.98	189.93	
114	2.8421	0.24581	-1.37082	314.21	201.81	
115	2.8174	-0.67856	0.67784	149.02	209.57	
116	2.7931	1.02301	-0.74311	259.00	189.68	
117	2.7692	-0.72290	0.39570	110.02	128.38	
118	2.7458	0.22591	-0.12335	10.73	103.77	
119	2.7227	-1.07452	-0.62128	249.57	154.77	

120	2.7000	0.07561	-0.72113	85.17	218.78	
121	2.6777	1.33779	-1.01403	456.50	303.24	
122	2.6557	0.10083	1.32108	284.38	254.24	
123	2.6341	-0.41618	-0.27642	40.44	138.49	
124	2.6129	-0.83233	-0.27596	124.57	117.25	
125	2.5920	0.88883	0.41199	155.48	130.13	
126	2.5714	-0.75408	-0.40724	118.99	102.15	
127	2.5512	-0.30385	0.06902	15.73	67.94	
128	2.5313	0.41211	-0.63411	92.65	82.98	
129	2.5116	-0.45761	0.73629	121.75	111.95	
130	2.4923	0.50825	0.68761	118.44	129.52	
131	2.4733	-0.94741	-0.40659	172.19	132.53	
132	2.4545	0.62585	0.19228	69.44	126.59	
133	2.4361	-0.53962	-0.86540	168.50	164.28	
134	2.4179	0.92457	0.86017	258.34	198.01	
135	2.4000	-0.79327	0.20517	108.76	215.40	
136	2.3824	-1.38513	0.62410	373.91	260.41	
137	2.3650	-0.23435	-1.18245	235.40	208.35	
138	2.3478	0.42819	-0.19990	36.18	97.01	
139	2.3309	0.18366	-0.23466	14.38	80.34	
140	2.3143	-1.11135	0.15983	204.22	147.22	
141	2.2979	-0.02814	-1.14225	211.49	150.46	
142	2.2817	0.21824	-0.07225	8.56	91.95	
143	2.2657	0.29782	0.80233	118.65	73.56	
144	2.2500	-0.38185	0.26241	34.78	68.15	
145	2.2345	-0.66160	-0.10187	72.59	88.08	
146	2.2192	0.72917	-0.73635	173.97	141.77	
147	2.2041	0.42738	-0.09683	31.11	341.25	
148	2.1892	0.55040	2.49823	1060.15	672.33	
149	2.1745	-2.00352	-0.83532	763.32	608.24	
150	2.1600	0.47837	-0.02525	37.18	254.71	
151	2.1457	0.23807	-0.50854	51.08	95.29	
152	2.1316	-0.48512	0.66519	109.81	130.78	
153	2.1176	0.50899	-1.19724	274.18	169.45	
154	2.1039	-0.31978	0.49713	56.60	132.62	
155	2.0903	0.70441	0.62199	143.06	112.01	
156	2.0769	-0.66649	0.21987	79.79	112.78	
157	2.0637	-0.09585	0.96764	153.17	125.27	
158	2.0506	-0.04562	-0.80095	104.26	140.89	
159	2.0377	1.07947	0.33955	207.45	162.12	

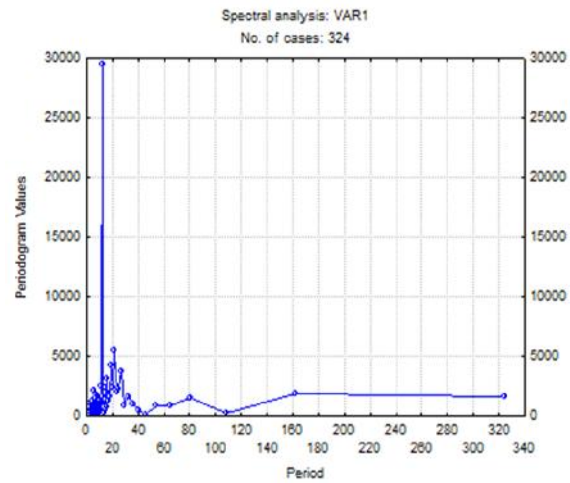

Figure 2. Time Domain Aspect of Periodogram

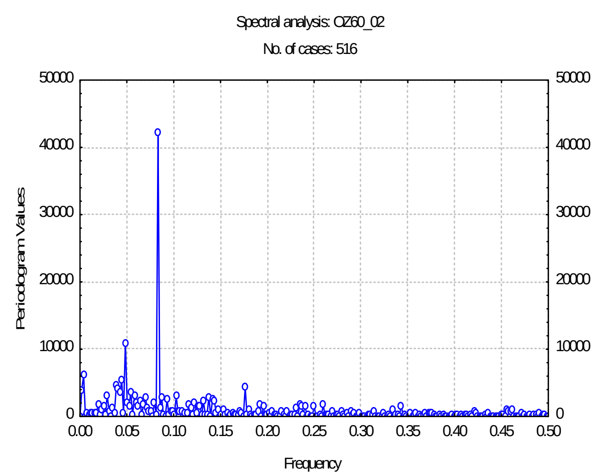

Figure 3. Frequency domain Aspect of Periodogranm

It has been noticed that we have power in each Fourier component, called power spectral density or more simply a power spectrum. Since the power is proportional to the square of the amplitude of an oscillation and since there are both sine and cosine terms 90^0 out of phase, the power S(f) at frequency m f_0 is

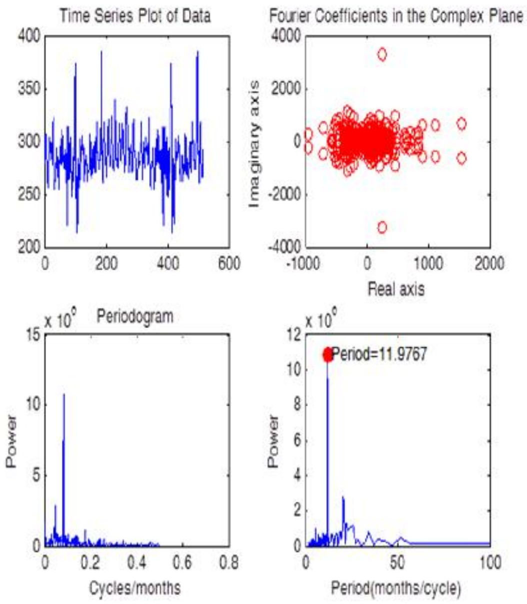

Figure 4. Plot Shows The Temporal Fluctuations, Power Vs Cycles /Months, Fourier Components In Complex Planes And Power Vs Period (Months / Cycle)

$$S_m = a^2_m + b^2_m$$

Only relative values of S_m are taken. Albeit it could be normalized by various procedures.

Fourier analysis is a mathematical technique used to obtain the frequency spectrum of an a-periodic signals, if the time domain representation is given. Similarly, Fourier series is a technique to decompose the entire series into components and Fourier transform is a technique to reduce the series into components. The periodicity in the ozone layer sequence is found using FFT that is equal to 11.98 as shown in the power spectrum of the ozone data set figure.4. Discrete time Signal processing techniques to collect, store and analyze the data in which Fourier transform, short time Fourier Transform (STFT) or frequency-time spectrograms are found as shown in the above figures.

3. Wavelet Analysis

The word wavelet was adopted to express the idea of small waves in an intuitive which is associated with localized waves like function whose values fluctuate in short period, in order to a function be called a wavelet. Wavelet analysis is a particular time-or space-scale representation of signals that has been found in atmospheric application like here in this case for the characterization of ozone concentration.

Wavelet is the name given to a set of mathematical functions used for image Compression, the actual wavelet analysis is a mathematical technique, which is very useful for numerical analysis and manipulation of multidimensional discrete signal sets. Originally applied in geophysics to the analysis of seismic signals, the wavelet transforms were better and broadly formalized thanks to mathematicians, physicists, and engineers efforts. In atmospheric applications, the main characteristic of wavelet technique is the introduction of the time frequency decomposition. When wavelet analysis is used to study a given signal, it is essential to choose the best wavelet representation for the signal under study. In statistical applications, the wavelet transform is linear and covariant under translation and dilatation transform. The wavelet transform can be used in the analysis of non-stationary signals to obtain information on the frequency or scale variations of those signals and to detect its structures localization in time and or in space.

Wavelet analysis is not replacing frequency analysis but it is rather an important refinement and expansion of it. The wave let can also be of great help in uncovering the presence and absence of certain frequencies in a physical system as OLD. Fourier analysis analyses a signal globally, whereas wavelet analysis looks into the signal locally.

4. Comparison between Fourier Transform and Wavelet

It is a fact that most of the signals including the ozone layer thickness cover a wide range of frequencies. Instead of transient components thee is a frequently a direct correlation between the characteristic frequency of a given segment of the ozone signal and time duration of that signal. Wavelet analysis has a number of advantages over Fourier analysis they are particularly attractive. Unlike the Fourier transform that generates record averaged values of amplitude and phase for each frequency component or harmonic,ω, the wavelet transform yields a localized, "instantaneous" estimate for the amplitude ans phase of each spectral component in the data set like ozone. This gives wavelet analysis an advantage in the analysis of signals like ozone concentration data set in which amplitudes and phases of the harmonic components are changing rapidly in time and space.

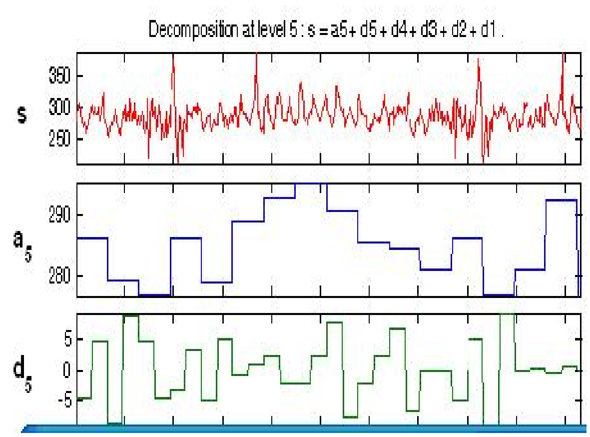

Figure 5. Haar Wavelet with Level 5, Full Decomposition at Level 5.

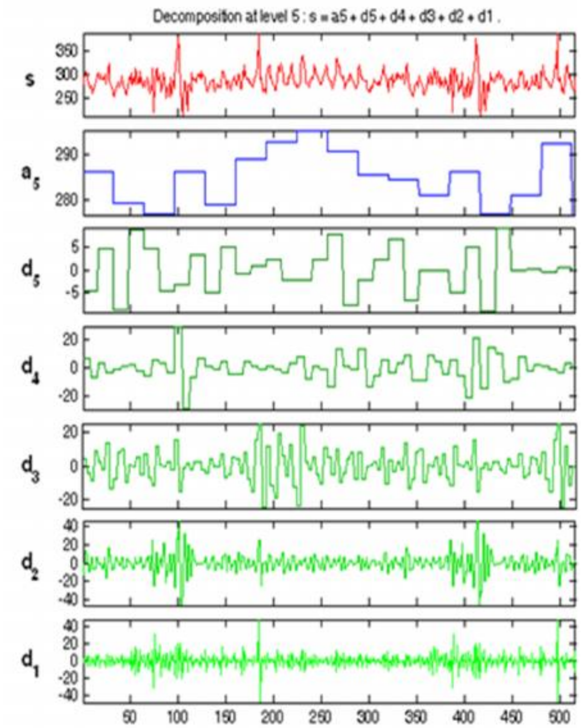

Figure 6. An Ozone Layer Signal Analyzed By Haar Wavelet. A Decomposition of Order 5. The Signal S Lives In V_0 And It Is Decomposed Into Its Approximation A_5 As A Member Of V_{-5} And Increasingly Finer Details D_j Is A Member Of W_{-J} , J =5,4,3,2,1.

Indeed the wavelets provide a framework for signal decomposition in the form of a sequence of signals known as approximation signals with decreasing resolution supplemented by a sequence of additional touches called details as illustrated in the ozone signal in figure 5 figure 6.

5. Conclusion

In this communication we have compared Fourier analysis and wavelet analysis and found that Wavelet analysis has a number of advantages over Fourier analysis they are particularly attractive. Unlike the Fourier transform that generates record averaged values of amplitude and phase for each frequency component or harmonic,ω, the wavelet transform yields a localized, “instantaneous” estimate for the amplitude ans phase of each spectral component in the data set like ozone.Wavelet transformed spectral analysis is used to find significance periodicities of the series. Also wavelet coherence analysis is used to find the different phases of the periodicities of the OLD data sets.

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