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Analysis and Synthesis of Electrocardiogram (ECG) using Fourier and Wavelet Transform

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Abstract

Electrocardiogram (ECG) is the study of the electrical signals of the human heart that are generated by the pumping action of the heart caused by the polarization and depolarization of the nodes of the heart. These signals must be interpreted with great accuracy and efficiency as they are paramount in prognosis and subsequent diagnosis of the condition of the patient. The goal of this project is to analyze the ECG signals following Fourier and Wavelet transforms, and to highlight and demonstrate the advantages of the Wavelet transform. Firstly, it involves simulating the temporal digital ECG signal and explaining the signal constituents, i.e., P, Q, R, S, T waves while staying in the time domain. Secondly, the ECG signal will be transferred into the frequency domain for quick, fast, and compressed analysis and carry out signal processing using Fourier analysis and highlight the pros and cons of this technique. Thirdly, wavelet analysis will be explored and demonstrated to mitigate the shortcoming of the former tool, i.e., Fourier. At this stage, various ECG signals, mimicking abnormalities, will be analyzed. This work will highlight the effectiveness of wavelet analysis as a tool to examine ECG signals. This work, hence, will entail, comparison of both transformation methods by utilizing the computational power of MATLAB.

Keywords: *ECG, Fourier transform, Hann window, Wavelet transform, Daubechies, Symlet*

1. Introduction

Ever since the inception of ECG at the turn of the 20th century, it has been an important tool for medical doctors to study and understand the functioning of the human heart. ECG has played an important role in helping practitioners to diagnose cardiac conditions and treat them accordingly. Currently, with an unprecedented rate of improvement of technology, ECG signals have been studied extensively. Numerous analysis and signal processing techniques have been employed for this purpose, ranging from various Fourier transform techniques like Fast Fourier Transform (FFT), zoom FFT to the more common Wavelet analysis. Apart from these methods, other techniques like Neural networks or differential equation procedures have been utilized successfully.

Fourier transform has been used for ECG signal synthesis for a long time now. The authors in [1], [2] utilized the Fourier series technique to generate the normal and abnormal ECG signals. However, no further analysis was performed in this work. Bennet et al. [3] came up with an interesting use of Fourier analysis of ECG vis-à-vis its shortcomings. The authors came up with a device to detect only two conditions, namely, tachycardia and bradycardia. These abnormalities depend only on the heart rate, which is easily measurable through FFT. Similarly, Lukáč and Ondráček [4] took advantage of this use of FFT and used it to calculate the heart rate.

Parak and Havlik [5] used statistical and differential mathematical tools to de-noise the ECG signal before utilizing it for making an implementable method for a real-time stress test. The proposed algorithm could work even in the presence of disturbance from the movement of muscles. Their main goal was to design a digital computing algorithm that could be implemented in real-time. Hence, the differential approach was very fast and effective. Murugan and Ramesh [6] used the zoom FFT as a less explored technique for analyzing ECG signals. They produced the ECG waveform using MATLAB code and then used the zoom FFT technique to detect the QRS complex and P and T peaks. The obtained results were compared with

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those obtained by conventional FFT practices. It concluded that the quality of spectrum for ECG analysis was better when using zoom FFT which in turn was better for diagnosing cardiac conditions and all this was obtained while not saturating the processor capabilities. In [7]–[29] the authors used Wavelet Transform techniques to analyze ECG signals. As a first step, the authors filtered the noise in ECG signals which were traced to various sources such as from the electric wires involved, and muscle activity. The use of the Daubechies family [30] of the wavelet transform was abundantly found in the literature. In [10], the author used *db2* of Daubechies family of Continuous Wavelet Transform (CWT) as it is suggested to provided better diagnostic ability. In [11], the author utilized *db6* of the Daubechies family of orthogonal wavelets, whereas in [13] the authors used *db10*. In all these works, the efficiency of using wavelets was highlighted. Castro et al. used an optimal mother wavelet technique in their work [31]. Rather than using a predefined wavelet family, the authors found out the wavelet that fits a specific ECG signal. Tamil et al. [32] used the Discrete Wavelet Transform (DWT), also discussed in [14], [18], [22], [27], for extraction of the characteristics of the ECG signal which was then fed to a hybrid neuro-fuzzy system consisting of Neural Networks and Fuzzy Logic. This method proved to be very accurate. However, due to the lack of an adequate database for various heart ailments, there is still room for improvement. The diagnostic ability, though, was increased considerably by using this hybrid system. Largely all the work harnessing the benefits of wavelet transform utilizes the coefficients, conversely, Peng and Wang [21] took a different approach where they employed the eigenvalues for detecting myocardial abnormalities [33] in the human through the recorded ECG signals. Daamouche et al. [23] classified the ECG signals using a polyphase representation of wavelet filter bank through a particle swarm optimization framework. The authors concluded that the proposed method was more effective than using standard wavelets like Daubechies and Symlet at the cost of far higher computational time.

The use of wavelet was not only limited to diagnosis rather was utilized even for matching the shape of a wavelet with the ECG signal [34]. Apart from the use of CWT and DWT, Cross Wavelet Transform was explored in the literature too [35]–[38] as well as their intermittency factor and energy percentage contribution within the signal [39]. However, the accuracy was not as high in this case as compared to the conventional wavelet techniques.

Most of the work focused on detecting heart diseases and cardiac conditions from the analyzed ECG signals, however, Sasikala and Wahidabanu [12], Mahmoud and Jusak [40] and Dar et al. [41] took the work further. They not only analyzed the ECG signal but also attempted to find a novel application of this analysis. They claimed that ECG signals, like fingerprints and retinal signatures, are unique to each individual and can be used as an identification tool. They presented analysis procedures to get this identification utility from these signals using Wavelet transform.

In this work, we aim to deconstruct an ECG signal using Fourier transform and a variety of orthogonal families of the Wavelet transform. These deconstructed waves will then be analyzed by time-shifting and stretching in the time domain. Following which we target to reconstruct the ECG signal using the obtained coefficients. This analysis will pave way for the synthesis of artificial heart signals and prognosis, the utility of which cannot be stressed enough in the modern day. All the work in the literature deals exclusively with only one of the two transforms. In this paper, we aim to provide a comparative study between Fourier and Wavelet transform and highlight the effectiveness of Wavelet Transform for ECG signal investigation.

2. ECG Signal

The ECG signal helps us study the condition of the human heart. It has certain characteristic features that give it meaning and helps medical practitioners understand the physiology of the patient's heart. It has proved to be a life-saving tool by aiding the diagnosis and prognosis of various heart ailments.

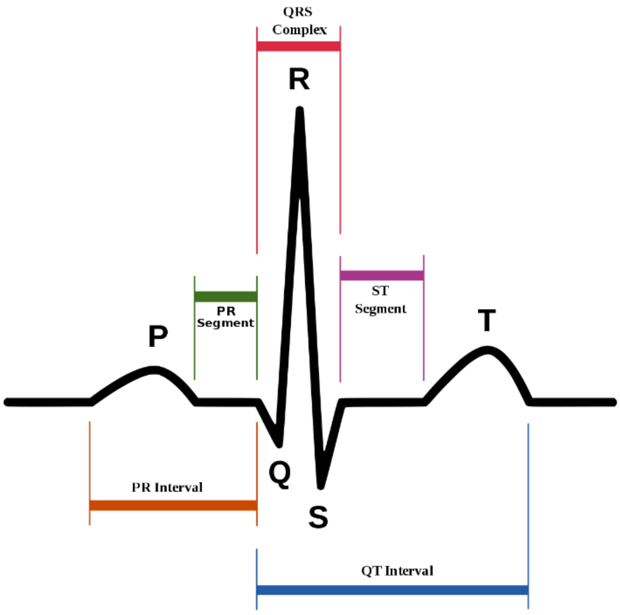

Fig. 1. Normal Sinus Rhythm

An electrocardiogram, as shown in Fig. 1 for a normal heartbeat, is composed of several 'waves' and 'segments' that are connected by an isoelectric line. Each wave and segment signify a particular action of the heart.

The first component is the 'P-wave' which indicates the depolarization of the sinoatrial node. This wave has a typical duration of $100\,ms$ and a peak value of $0.3\,mV$. The most distinguishing trait of the ECG signal is the 'QRS complex'. This complex is made up of 3 waves, Q-, R-, and S- waves. The Q-wave and S-wave are negative parts and R-wave is the highest peak in the ECG signal. The 'QRS complex' has a time duration between 50 to $110\,ms$. The final wave is the 'T-wave' which shows the process of repolarization of the heart. The heart returns to its idle state during this wave. This 'T-wave' has an amplitude of around $0.8\,mV$ and lasts around $0.42\,ms$.

Apart from the waves, another important part of the ECG signal is the connecting intervals. These segments are isoelectric components, i.e., the voltage remains at $0\,mV$ during these intervals. The 2 intervals of interest are 'P-R interval' and 'S-T interval'.

The ECG signals used in this work are obtained from the PhysioNet master database [42] which contains modified copies of 3 PhysioNet databases [42]–[44]. This contains pre-filtered Normal Sinus Rhythm, various arrhythmia signals, and congestive heart failure records. For analysis, only the initial 10 seconds of the data is considered. Since the signal is sampled at $128\,Hz$, it gives enough data points (1280) in the span of 10 seconds to effectively evaluate the signal without much computational burden. The signal segment used is shown in Fig. 2. The peak detection is a very important phenomena related to ECG signal. It gives us a measure of the heart rate which is the basis of many pathological condition detection. The peak detection includes primarily includes detecting the 'R-wave'. The time interval between 2 consecutive 'R-waves' helps us calculate heart rate of the patient.

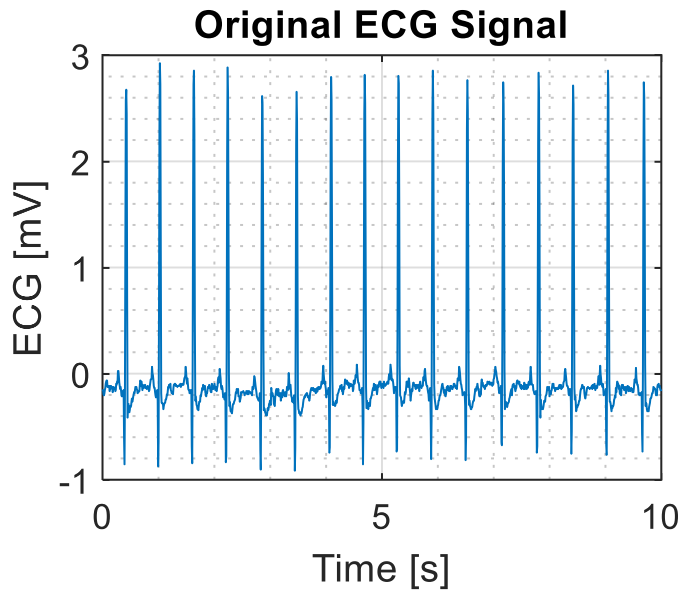

Fig. 2 Filtered ECG signal segment

The heart rate of the sample shown in Fig. 1 was 95 beats per minute (bpm), and the value obtained through the peak detecting algorithm was 96 bpm. Furthermore, peak detection also includes detecting all the waves in the ECG signal. In Fig. 3 we see that all the waves are characterized by their crests and troughs.

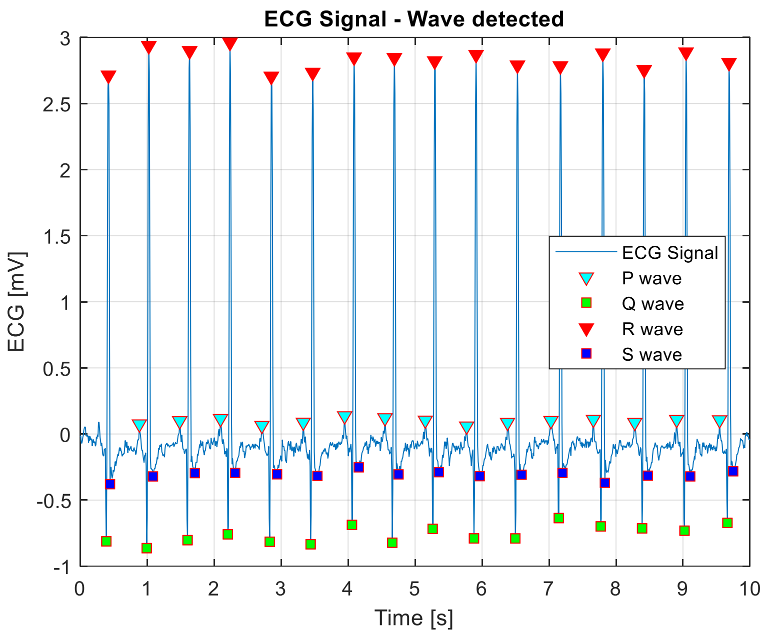

Fig. 3 ECG signal peak and wave detection

3. Fourier Transform

In this section, we will be discussing the use of Fourier transform for ECG analysis. Fourier transform is a powerful tool for analyzing stationary signals. The frequency-domain analysis gives a lot of information about the signal. However, when non-stationary signals, like ECG, are to be analyzed, Fourier transform falls short. Small changes in the heart rhythm most likely will go undetected if analyzed through Fourier transform. We perform Fast Fourier Transform (FFT) on the ECG signal. This gives us information about the High Frequency (HF) and

Low Frequency (LF) components of the ECG. The LF gives information about the physiological activities of the heart whereas HF indicates respiratory activity.

To analyze the signal using Fourier Transform, we perform FFT on the ECG signal. This gives us the frequency domain response of the signal.

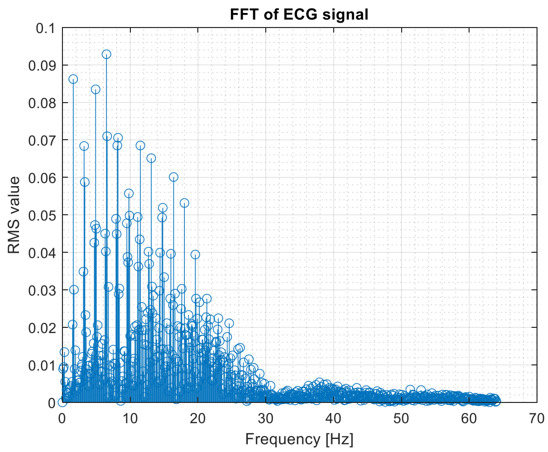

Fig. 4 FFT of ECG signal

We see in Fig. 4 that many frequency components are needed to characterize an ECG signal. This becomes clear when we plot an envelope curve of the peaks of FFT response.

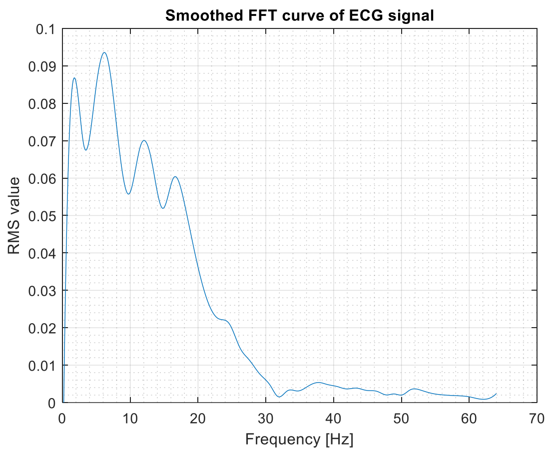

Fig. 5 Peak envelope of FFT

The response shown in Fig. 5 corresponds to a broadband response, re-iterating the point that several data points are needed to recreate the ECG signal using Fourier Transform.

FFT often falls short in accurately extracting data from non-stationary data and is not a very efficient approach. To examine this issue further, we tried to use the Hann smoothing method using a moving average window. In this method, we created 'Hann' windows of 1 *s* time duration with overlap to cover the entire sample signal length as shown in Fig. 6.

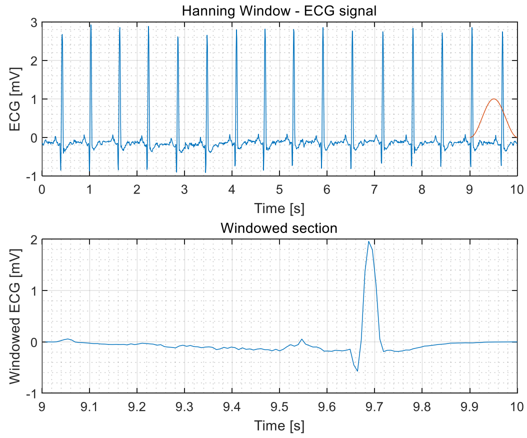

Fig. 6 Hanning of ECG signal

Single-sided FFT was performed on this windowed signal and an average was taken to investigate the frequency response of the ECG signal depicted in Fig. 7. It can be seen that despite taking an average of 19 windows, the ECG frequency response is broadband. This implies that it is not useful to analyze an ECG signal using FFT.

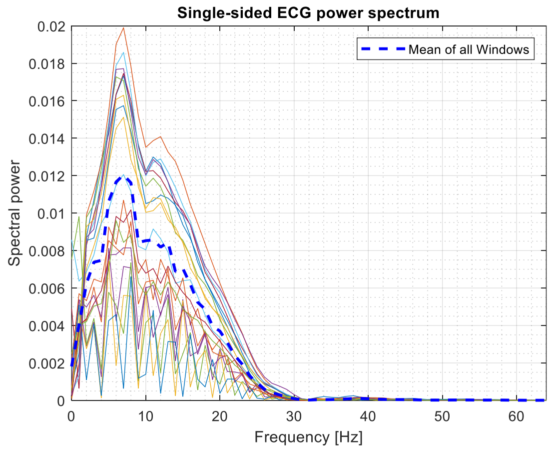

Fig. 7 ECG Power Spectrum for various Hann windows

This idea is supported by trying to recreate ECG for normal sinus rhythm using an 8-term Fourier series. The signal synthesized by Fourier series is compared with the original ECG signal in Fig. 8.

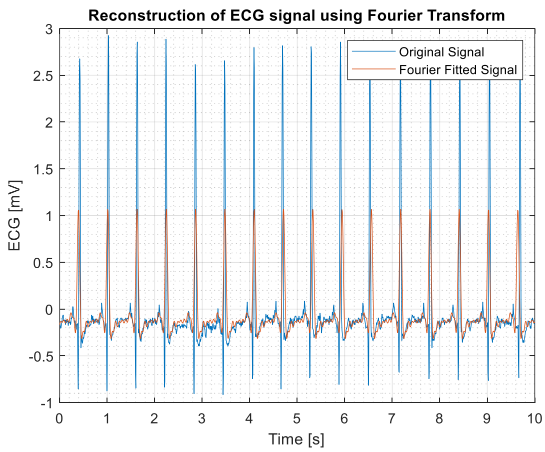

Fig. 8 Fourier transform reconstruction

The Fourier model used to generate the Fourier series is given in Eq. (1). The corresponding coefficient values are given in Table 1 which are within a 95% confidence bound.

$$f(x) = a_0 + a_1\cos(xw) + b_1\sin(xw) + a_2\cos(2xw) + b_2 sin(2xw) + a_3 cos(3xw) + b_3 sin(3xw) + a_4 cos(4xw) + b_4 sin(4xw) + a_5 cos(5xw) + b_5 sin(5xw) + a_6 cos(6xw) + b_6 sin(6xw) + a_7 cos(7xw) + b_7 sin(7xw) + a_8 cos(8xw) + b_8 sin(8xw) \quad (1)$$

Table 1 Fourier series coefficients

Coefficients	Value	Coefficients	Value
a_0	-0.4492	w	10.19
a_1	-0.1526	b_1	-0.1114
a_2	-0.02265	b_2	0.1919
a_3	0.1812	b_3	-0.08022
a_4	0.1707	b_4	-0.08524
a_5	0.03242	b_5	0.1387
a_6	0.05021	b_6	-0.07768
a_7	-0.06914	b_7	0.00465
a_8	0.03006	b_8	0.03388

It is evident that the Fourier series representation fails to replicate the peaks of the ECG signal. Due to this, important information maybe lost and hence, Fourier transform falls short in analyzing ECG signals.

4. Wavelet Transform

In this section, we will utilize Wavelet transform to analyze ECG signals. Subsequently, we will discuss its application in clinical prognosis. Wavelet transform is a potent means for analyzing non-stationary signals. The most important feature of the Wavelet transform is that it retains the time-domain information of the signal while also enabling the analysis in frequency-domain. This is paramount in the assessment of non-stationary waves.

Wavelet transform is of two types, Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). Both these transforms can be used effectively for diagnosis using ECG signals.

4.1. Continuous Wavelet Transform

The CWT of a function $f(t)$ is obtained by the following equation:

$$W_c(b,a) = |a|^{\frac{1}{2}} \int_{-\infty}^{\infty} f(t)\psi^*(\frac{t-b}{a})\, dt \quad (2)$$

Where, $a, b \in R, a \neq 0$ are the scaling and shifting coefficients of the mother wavelet denoted by $\psi(t)$ respectively. The mean of a wavelet signal is zero, implying that the net area of the mother wavelet is zero.

CWT gives the spectrogram of the ECG signal. This helps us understand the signal effortlessly. It clearly shows the difference between normal and abnormal heart activity. It is seen in Fig. 9 the obvious difference between the two ECG signals. It gives medical practitioners a distinct image to quickly analyze the problem of the patient.

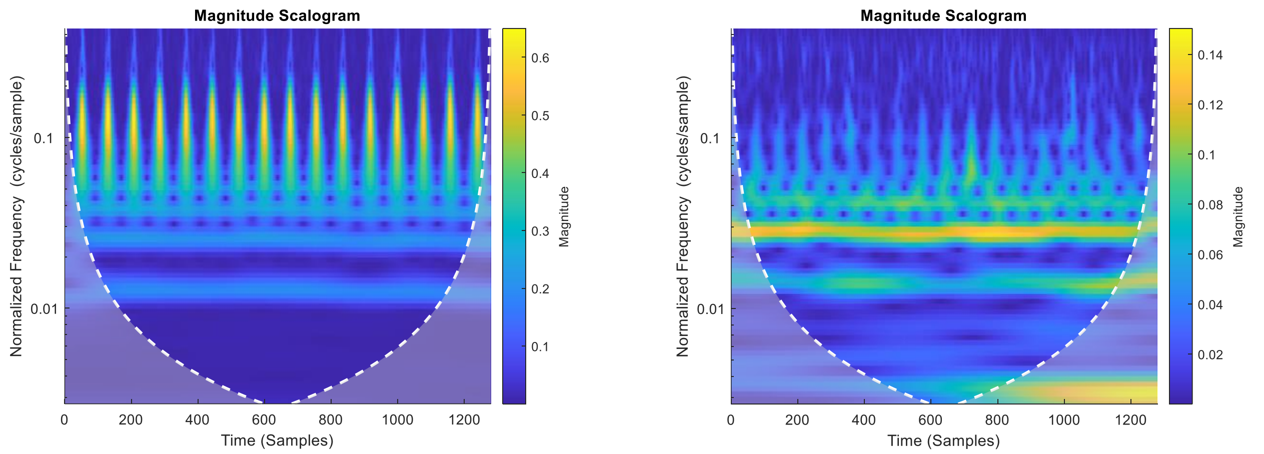

Fig. 9 CWT response a) Normal Sinus Rhythm b) Congestive heart failure

4.2. Discrete Wavelet Transform

In practice, when computers are used for implementing CWT then it must be in a discrete form giving rise to DWT. Being continuous causes redundancy in CWT. This problem is ably addressed by sampling CWT function in a dyadic grid. Hence, DWT is obtained by convoluting the signal with the orthonormal dyadic wavelet function and the scaling function. The dyadic grid is [45]:

$$a = 2^{-m} \text{ and } b = n2^{-m} \quad (3)$$

where $m, n \in Z$.

From (2) and (3) we obtain the DWT function as:

$$W_d(m,n) = \int_{-\infty}^{\infty} f(t)\, \psi_{m,n}^*(t) dt \quad (4)$$

As an orthonormal wavelet basis is used there is no redundancy. Furthermore, we obtain a Multi-Resolution Analysis (MRA) system, which decomposes the original ECG signal into scales of different frequency and time resolution. The fundamental concept involved in MRA is to find the average features and the details of the signal via scalar products with scaling signals and wavelets. Using these techniques, the ECG is disintegrated using an optimal mother wavelet, the wavelet that most closely resembles the shape of the original signal. The decomposition includes separating the signal into high frequency and low-frequency components. This is done by decomposing the ECG signals in many levels of approximate and detailed coefficients. The detailed coefficients are obtained from the high-frequency component of the wavelet function. The approximate coefficients, given in Eq. (5) at scale m and location n, have the details of the scaling functions ($\phi(t)$) and are low frequency components of the signal. These approximate coefficients are further broken down based on the number of levels the signal is to be decomposed into. When the ECG signal is broken into approximate and detailed coefficients based on the frequency, it still retains the time-domain information. This not only helps us to understand what the abnormalities in the ECG signal are but also helps us find the exact point at which activity of the heart happens. This decomposed signal can then be reconstructed using the obtained coefficients without much loss of information. The wavelet decomposition and the reconstruction follow the steps as depicted in Fig. 10.

$$S_{m,n} = \int_{-\infty}^{\infty} f(t)\,\phi_{m,n}(t)dt \tag{5}$$

The discrete approximation of the original signal is given by:

$$f_0(t) = f_M(t) + \sum_{m=1}^{M} d_m(t) \tag{6}$$

Where f_M is the mean signal approximation at scale M given by $f_M(t) = S_{M,n}\phi_{M,n}(t)$ and d_M is the detail signal approximation at scale m, given by $d_m(t) = \sum_{n=0}^{M-m} T_{m,n}\psi_{m,n}(t)$.

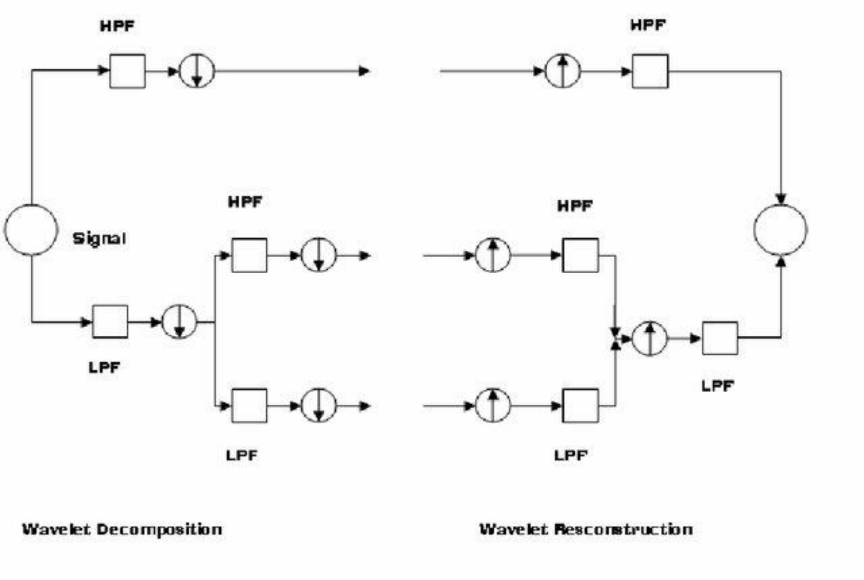

Fig. 10 Two-level Wavelet decomposition and reconstruction [46]

From Fig. 10 we see that the approximation of the signal at a given scale is the combination of the approximate and detail at the next smaller scale given in Eq.

$$f_m(t) = f_{m-1}(t) - d_m(t) \tag{7}$$

In this work, we tried various orthogonal wavelets and decided to explore the use of Symlet wavelet, a modified version of the more popular Daubechies wavelet. This wavelet was chosen as $sym4$ of the Symlet family had a shape very similar to the original normal ECG signal as shown in Fig. 11. We generated the ECG signal recreated using the aforementioned wavelet and compared it with the original ECG signal to gauge the effectiveness of the wavelet as an analysis tool for ECG signals. The plot in Fig. 12 shows that the reconstructed signal traces the normal ECG signal with great accuracy. This establishes the effectiveness of Wavelet Transform in analyzing and synthesizing ECG signals.

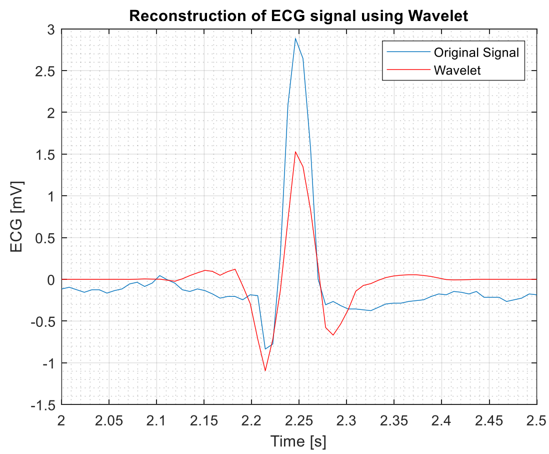

Fig. 11 Matching wavelet with ECG signal

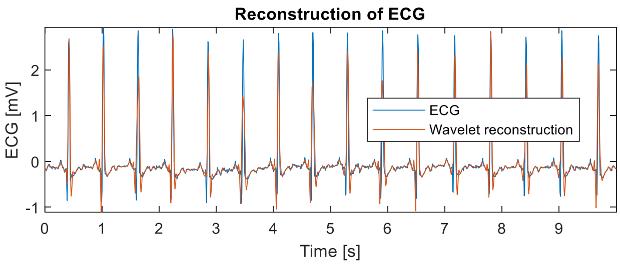

Fig. 12 Wavelet reconstruction of normal ECG signal

4.3. Wavelet Analysis

Wavelet analysis gives the details about the ECG signal in both the time and frequency domain. Also, wavelet decomposition gives different coefficients for different signals, implying, for all different arrhythmias and congestive heart failures, the coefficients remain distinct. This plays a major role in distinguishing the ECG and consequently in diagnosing the patient.

In this paper, we compare the decomposition coefficients are the 2 levels of the normal sinus rhythm with arrhythmias and with congestive heart failure records. Based on the comparison we will be able to classify the ECG records as arrhythmias or heart failure. This should help in a quick analysis of the patient's condition. From the plots in Fig. 13 to Fig. 22, we see that the coefficients of the normal sinus rhythm are significantly different when compared to various diseases. In each of the following figures, the normal sinus rhythm's coefficients, both decomposition, and reconstruction are compared with that of several abnormalities. In Fig. 13 and Fig. 14 the comparison gives is for hyperkalemia where the 'P-wave' is missing, and the 'T-wave' has a high magnitude. Furthermore, we compared the myocardial ischemia shown in Fig. 15 and Fig. 16, which has an inverted 'T-wave', with a normal heartbeat. Two very common arrhythmias are bradycardia, depicted in Fig. 17 and Fig. 18, where the heart rate drops below 50 bpm, and tachycardia, exhibited in Fig. 19 and Fig. 20 a case in which the heart rate exceeds 120 bpm. In Fig. 21 and Fig. 22, we see the case of congestive heart failure.

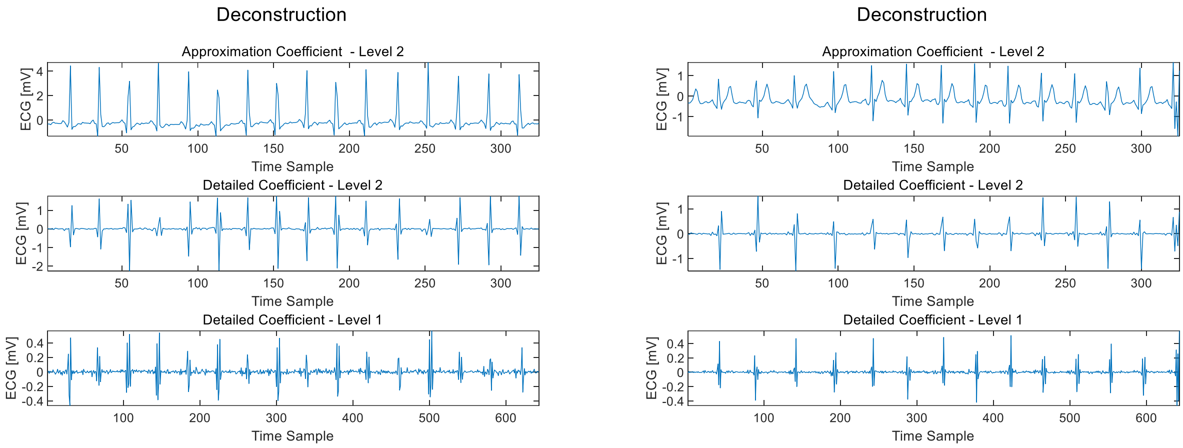

Fig. 13 Deconstruction coefficients a) Normal Sinus Rhythm b) Hyperkalemia

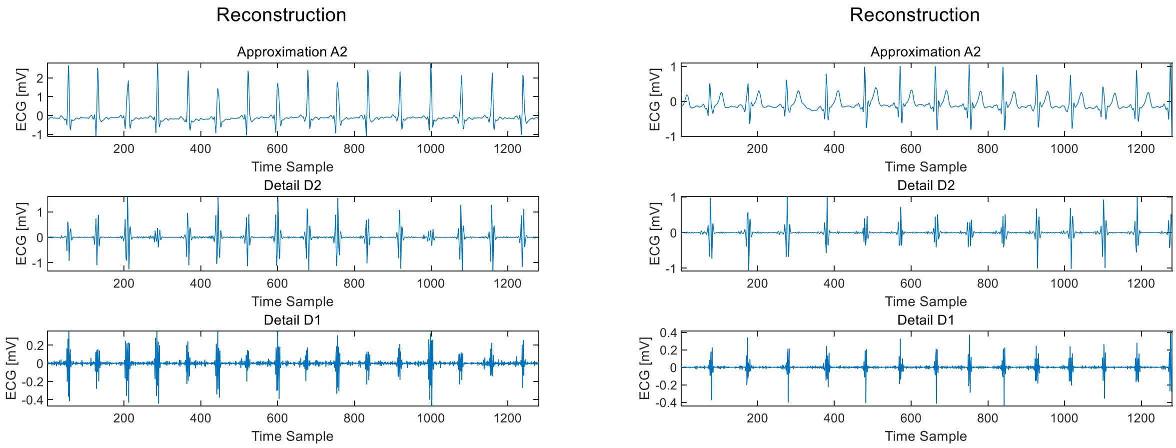

Fig. 14 Reconstruction coefficients a) Normal Sinus Rhythm b) Hyperkalemia

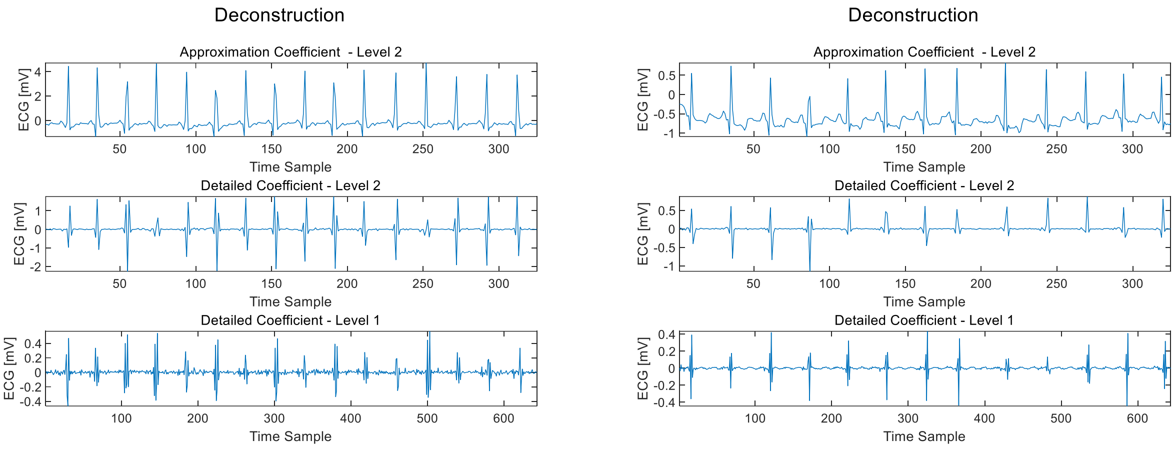

Fig. 15 Deconstruction coefficients a) Normal Sinus Rhythm b) Myocardial Ischemia

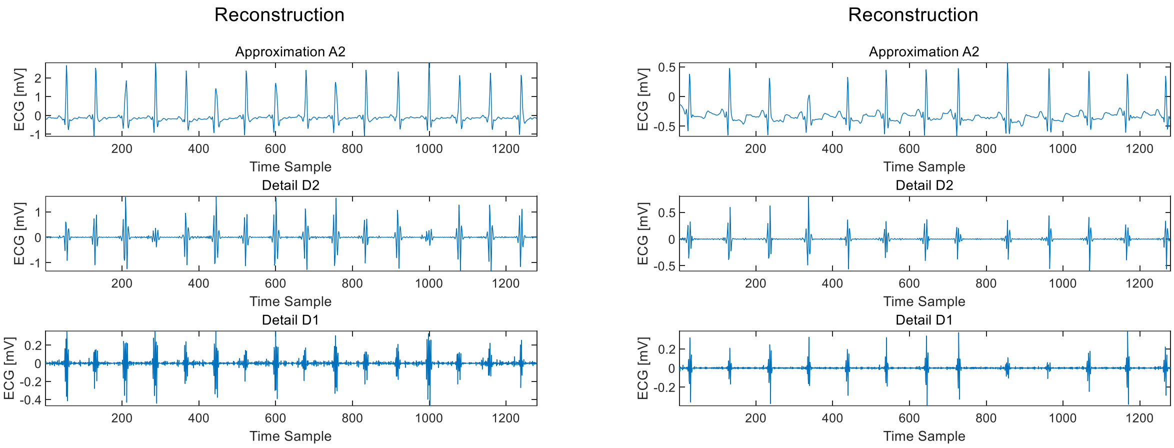

Fig. 16 Reconstruction coefficients a) Normal Sinus Rhythm b) Myocardial Ischemia

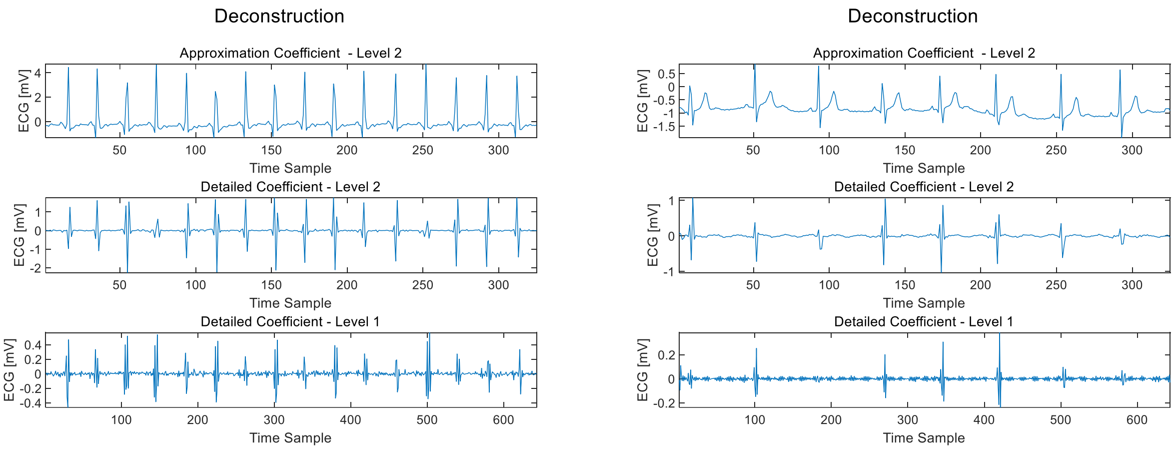

Fig. 17 Deconstruction coefficients a) Normal Sinus Rhythm b) Bradycardia

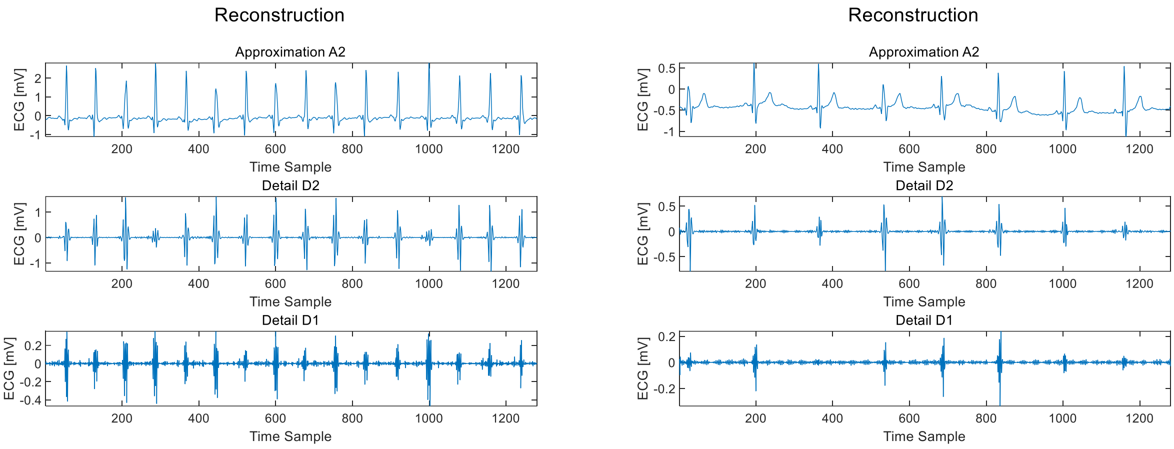

Fig. 18 Reconstruction coefficients a) Normal Sinus Rhythm b) Bradycardia

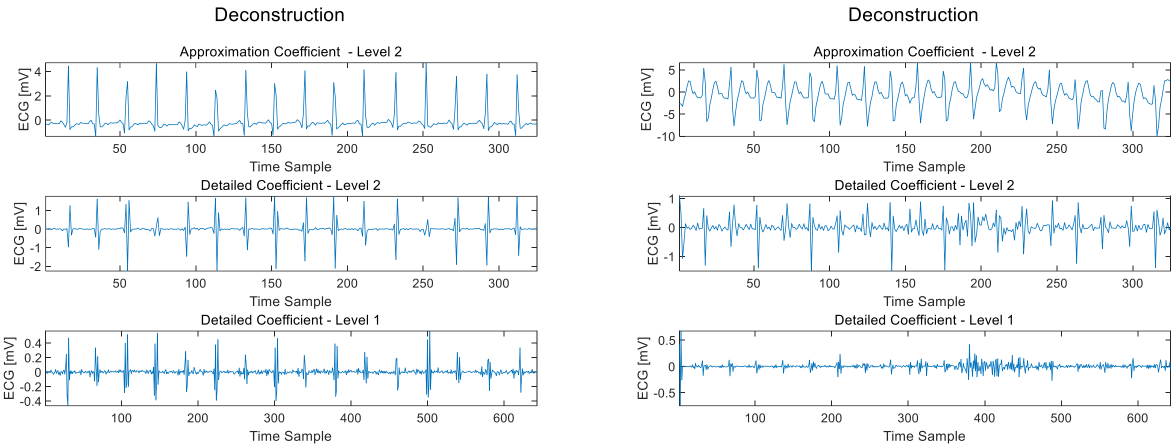

Fig. 19 Deconstruction coefficients a) Normal Sinus Rhythm b) Tachycardia

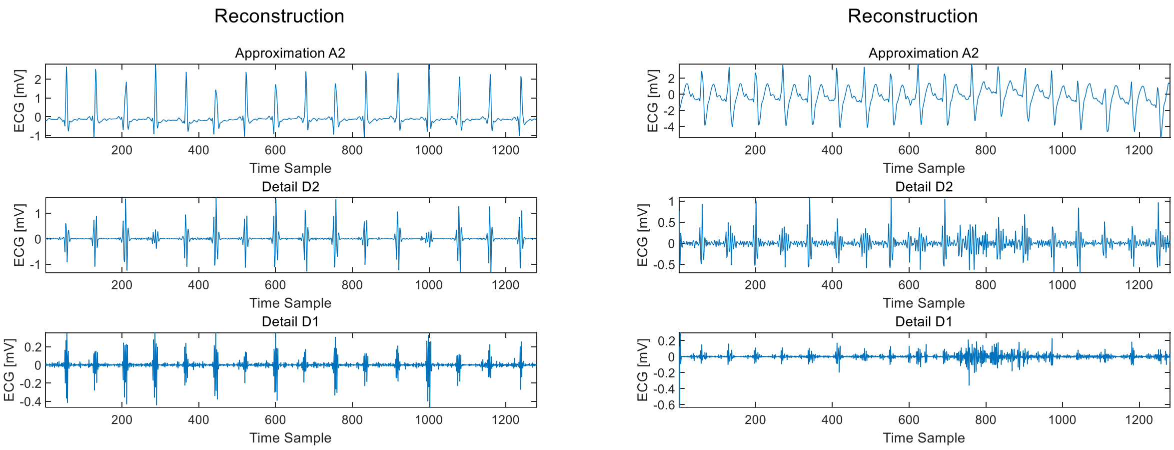

Fig. 20 Reconstruction coefficients a) Normal Sinus Rhythm b) Tachycardia

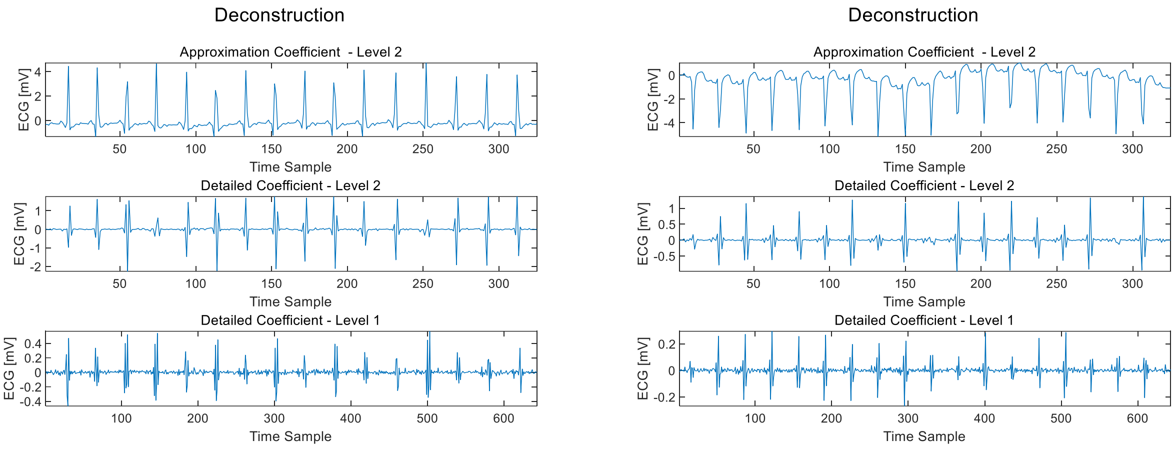

Fig. 21 Deconstruction coefficients a) Normal Sinus Rhythm b) Congestive heart failure

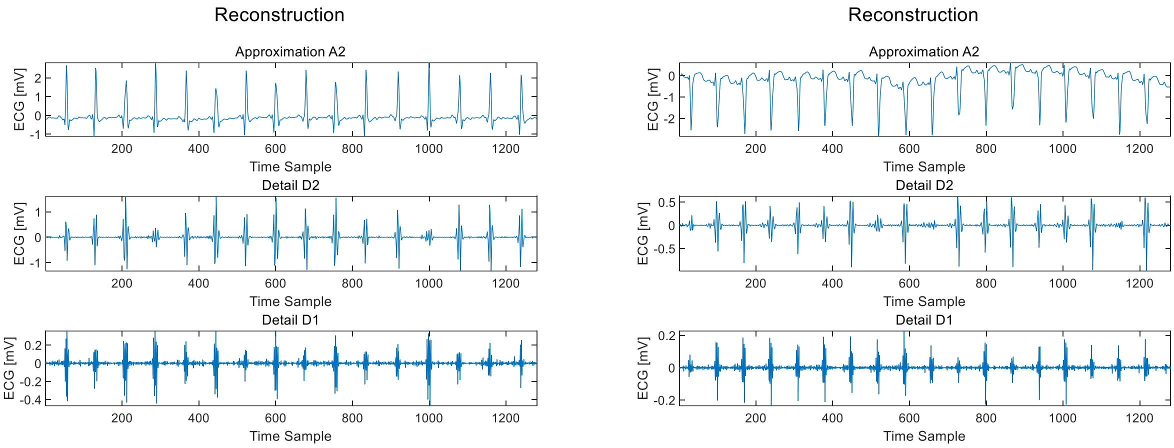

Fig. 22 Reconstruction coefficients a) Normal Sinus Rhythm b) Congestive heart failure

5. Conclusion

To begin with, we studied the effectiveness of the Fourier Transform in analyzing ECG signals. It was seen that it was not very effective and could not give full information on the ECG signal because it analyzes a signal only in the frequency-domain. In that way, the time localization information is lost. Hann window technique was deployed too for extracting more information using FFT. However, even this technique failed as was seen when we tried to reconstruct the ECG wave using the coefficients obtained from the aforementioned tools.

This work was then advanced to study ECG signals using Wavelet Transform. Wavelet has been an efficient tool for analyzing non-stationary signals like ECG. Firstly, we used the CWT method for analyzing the heartbeat signals. It was seen that this provided an uncomplicated way to understand the signal by giving a spectrographic representation of the signal, in which any abnormalities were pronounced. Additionally, we investigated the use of DWT for this purpose. We saw that this was very effective as it addressed the shortcomings in Fourier analysis effectively. This was primarily due to the ability of wavelet analysis to analyze a signal in both time- and frequency-domain. This helped us understand the exact abnormality at the exact instant in time. This is paramount in diagnosis. From this, we can conclude that wavelet transform was superior to Fourier in terms of examining the ECG signal. This implies that Wavelet Transform can be an effective clinical tool to analyze ECG signals and accurately diagnose heart conditions.

Nomenclature

a_n	Fourier series coefficient
b_n	Fourier series coefficient
$W_c(b,a)$	Continuous Wavelet Transform function
$\psi(t)$	Mother wavelet
$\psi^*(t)$	Complex conjugate of the mother wavelet
$W_d(t)$	Continuous Wavelet Transform function
$\phi(t)$	Scaling function

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